Trigonometric Functions

1. Prove that $\frac{\tan A+\sec A-1}{\tan A-\sec A+1}=\frac{1+\sin A}{\cos A}$

2. If $\frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha}=y$, then prove that $\frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}$ is also equal to y.

   $\left[\textbf{Hint}: \text{Express } \frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}=\frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}\cdot\frac{1+\cos\alpha+\sin\alpha}{1+\cos\alpha+\sin\alpha}\right]$

3. If $m\sin\theta = n\sin(\theta+2\alpha)$, then prove that $\tan(\theta+\alpha)\cot\alpha=\frac{m+n}{m-n}$

   [**Hint:** Express $\frac{\sin(\theta+2\alpha)}{\sin\theta}=\frac{m}{n}$ and apply componendo and dividendo]

4. If $\cos(\alpha+\beta)=\frac{4}{5}$ and $\sin(\alpha-\beta)=\frac{5}{13}$, where α lie between 0 and $\frac{\pi}{4}$, find the value of $\tan 2\alpha$ [**Hint:** Express $\tan 2\alpha$ as $\tan(\alpha+\beta+\alpha-\beta)$]

5. If $\tan x=\frac{b}{a}$, then find the value of $\sqrt{\frac{a+b}{a-b}}+\sqrt{\frac{a-b}{a+b}}$

6. Prove that $\cos\theta\cos\frac{\theta}{2}-\cos 3\theta\cos\frac{9\theta}{2}=\sin 7\theta\sin 8\theta$.

   [Hint: Express L.H.S. $=\frac{1}{2}[2\cos\theta\cos\frac{\theta}{2}-2\cos 3\theta\cos\frac{9\theta}{2}]$]

7. If $a\cos\theta+b\sin\theta=m$ and $a\sin\theta-b\cos\theta=n$, then show that $a^2+b^2=m^2+n^2$

8. Find the value of $\tan 22°30'$.

   $\left[\text{Hint: Let } \theta=45°, \text{ use } \tan\frac{\theta}{2}=\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}=\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}=\frac{\sin\theta}{1+\cos\theta}\right]$

9. Prove that $\sin 4A = 4\sin A\cos^3 A - 4\cos A\sin^3 A$.

10. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, then prove that $m^2 - n^2 = 4\sin\theta\tan\theta$
[Hint: $m + n = 2\tan\theta$, $m - n = 2\sin\theta$, then use $m^2 - n^2 = (m + n)(m - n)$]

11. If $\tan(A + B) = p$, $\tan(A - B) = q$, then show that $\tan 2A = \frac{p+q}{1-pq}$

[Hint: Use $2A = (A + B) + (A - B)$]

12. If $\cos\alpha + \cos\beta = 0 = \sin\alpha + \sin\beta$, then prove that $\cos 2\alpha + \cos 2\beta = -2\cos(\alpha + \beta)$.
[Hint: $(\cos\alpha + \cos\beta)^2 - (\sin\alpha + \sin\beta)^2 = 0$]

13. If $\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$, then show that $\frac{\tan x}{\tan y} = \frac{a}{b}$ [**Hint:** Use Componendo and Dividendo].

14. If $\tan\theta = \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$, then show that $\sin\alpha + \cos\alpha = \sqrt{2}\cos\theta$.

[Hint: Express $\tan\theta = \tan(\alpha - \frac{\pi}{4}) \Rightarrow \theta = \alpha - \frac{\pi}{4}$]

15. If $\sin\theta + \cos\theta = 1$, then find the general value of θ.
16. Find the most general value of θ satisfying the equation $\tan\theta = -1$ and $\cos\theta = \frac{1}{\sqrt{2}}$.

17. If $\cot\theta + \tan\theta = 2\operatorname{cosec}\theta$, then find the general value of θ.
18. If $2\sin^2\theta = 3\cos\theta$, where $0 \le \theta \le 2\pi$, then find the value of θ.

19. If $\sec x \cos 5x + 1 = 0$, where $0 < x \le \frac{\pi}{2}$, then find the value of x.

20. If $\sin(\theta + \alpha) = a$ and $\sin(\theta + \beta) = b$, then prove that $\cos 2(\alpha - \beta) - 4ab\cos(\alpha - \beta) = 1 - 2a^2 - 2b^2$ [Hint: Express $\cos(\alpha - \beta) = \cos((\theta + \alpha) - (\theta + \beta))$]

21. If $\cos(\theta + \phi) = m\cos(\theta - \phi)$, then prove that $\tan\theta = \frac{1-m}{1+m}\cot\phi$.

[Hint: Express $\frac{\cos(\theta+\phi)}{\cos(\theta-\phi)}=\frac{m}{1}$ and apply Componendo and Dividendo]

22. Find the value of the expression

$3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi+\alpha)\right]-2\left\{\sin^6\left(\frac{\pi}{2}+\alpha\right)+\sin^6(5\pi-\alpha)\right]$

23. If $a\cos 2\theta + b\sin 2\theta = c$ has α and β as its roots, then prove that

$\tan\alpha + \tan\beta = \frac{2b}{a+c}$.

[Hint: Use the identities $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$ and $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$].

24. If $x = \sec\phi - \tan\phi$ and $y = \operatorname{cosec}\phi + \cot\phi$ then show that $xy + x - y + 1 = 0$
[Hint: Find $xy + 1$ and then show that $x - y = -(xy + 1)$]

25. If θ lies in the first quadrant and $\cos\theta = \frac{8}{17}$, then find the value of

$\cos(30^\circ + \theta) + \cos(45^\circ - \theta) + \cos(120^\circ - \theta)$.

26. Find the value of the expression $\cos^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}+\cos^4\frac{5\pi}{8}+\cos^4\frac{7\pi}{8}$

[Hint: Simplify the expression to $2\left(\cos^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}\right)$

$=2\left[\left(\cos^2\frac{\pi}{8}+\cos^2\frac{3\pi}{8}\right)^2-2\cos^2\frac{\pi}{8}\cos^2\frac{3\pi}{8}\right]$

27. Find the general solution of the equation $5\cos^2\theta + 7\sin^2\theta - 6 = 0$
28. Find the general solution of the equation
$\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$
29. Find the general solution of the equation $(\sqrt{3}-1)\cos\theta + (\sqrt{3}+1)\sin\theta = 2$

[Hint: Put $\sqrt{3}-1 = r\sin\alpha$, $\sqrt{3}+1 = r\cos\alpha$ which gives $\tan\alpha = \tan\left(\frac{\pi}{4}-\frac{\pi}{6}\right)$
$\Rightarrow \alpha = \frac{\pi}{12}$]

Choose the correct answer from the given four options in the Exercises 30 to 59 (M.C.Q.).

30. If $\sin\theta + \operatorname{cosec}\theta = 2$, then $\sin^2\theta + \operatorname{cosec}^2\theta$ is equal to

(A) 1 (B) 4

(C) 2 (D) None of these

31. If $f(x) = \cos^2 x + \sec^2 x$, then

(A) $f(x) < 1$ (B) $f(x) = 1$

(C) $2 < f(x) < 1$ (D) $f(x) \geq 2$

[Hint: A.M $\geq$ G.M.]

32. If $\tan\theta = \frac{1}{2}$ and $\tan\phi = \frac{1}{3}$, then the value of $\theta + \phi$ is

(A) $\frac{\pi}{6}$ (B) π (C) 0 (D) $\frac{\pi}{4}$

33. Which of the following is not correct?

(A) $\sin\theta = -\frac{1}{5}$ (B) $\cos\theta = 1$

(C) $\sec\theta = \frac{1}{2}$ (D) $\tan\theta = 20$

34. The value of $\tan 1° \tan 2° \tan 3° \ldots \tan 89°$ is

(A) 0 (B) 1

(C) $\frac{1}{2}$ (D) Not defined

35. The value of $\frac{1-\tan^2 15°}{1+\tan^2 15°}$ is

(A) 1 (B) $\sqrt{3}$ (C) $\frac{\sqrt{3}}{2}$ (D) 2

36. The value of $\cos 1° \cos 2° \cos 3° \ldots \cos 179°$ is

(A) $\frac{1}{\sqrt{2}}$ (B) 0 (C) 1 (D) –1

37. If $\tan\theta = 3$ and θ lies in third quadrant, then the value of $\sin\theta$ is

(A) $\frac{1}{\sqrt{10}}$ (B) $-\frac{1}{\sqrt{10}}$ (C) $\frac{-3}{\sqrt{10}}$ (D) $\frac{3}{\sqrt{10}}$

38. The value of $\tan 75° - \cot 75°$ is equal to

(A) $2\sqrt{3}$ (B) $2+\sqrt{3}$ (C) $2-\sqrt{3}$ (D) 1

39. Which of the following is correct?

(A) $\sin 1° > \sin 1$ (B) $\sin 1° < \sin 1$

(C) $\sin 1° = \sin 1$ (D) $\sin 1° = \frac{\pi}{18°} \sin 1$

[Hint: 1 radian $= \frac{180°}{\pi} = 57°30'$ approx]

40. If $\tan\alpha = \frac{m}{m+1}$, $\tan\beta = \frac{1}{2m+1}$, then $\alpha + \beta$ is equal to

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{4}$

41. The minimum value of $3\cos x + 4\sin x + 8$ is

(A) 5 (B) 9 (C) 7 (D) 3

42. The value of $\tan 3A - \tan 2A - \tan A$ is equal to

(A) $\tan 3A \tan 2A \tan A$

(B) $-\tan 3A \tan 2A \tan A$

(C) $\tan A \tan 2A - \tan 2A \tan 3A - \tan 3A \tan A$

(D) None of these

43. The value of $\sin(45° + \theta) - \cos(45° - \theta)$ is

(A) $2\cos\theta$ (B) $2\sin\theta$ (C) 1 (D) 0

44. The value of $\cot\left(\frac{\pi}{4}+\theta\right)\cot\left(\frac{\pi}{4}-\theta\right)$ is

(A) -1 (B) 0 (C) 1 (D) Not defined

45. $\cos 2\theta \cos 2\phi + \sin^2(\theta - \phi) - \sin^2(\theta + \phi)$ is equal to

(A) $\sin 2(\theta + \phi)$ (B) $\cos 2(\theta + \phi)$

(C) $\sin 2(\theta - \phi)$ (D) $\cos 2(\theta - \phi)$

[Hint: Use $\sin^2 A - \sin^2 B = \sin(A + B)\sin(A - B)$]

46. The value of $\cos 12° + \cos 84° + \cos 156° + \cos 132°$ is

(A) $\frac{1}{2}$ (B) 1 (C) $-\frac{1}{2}$ (D) $\frac{1}{8}$

47. If $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$, then $\tan (2A + B)$ is equal to

(A) 1 (B) 2 (C) 3 (D) 4

48. The value of $\sin\frac{\pi}{10}\sin\frac{13\pi}{10}$ is

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) $-\frac{1}{4}$ (D) 1

[Hint: Use $\sin 18° = \frac{\sqrt{5}-1}{4}$ and $\cos 36° = \frac{\sqrt{5}+1}{4}$]

49. The value of $\sin 50° - \sin 70° + \sin 10°$ is equal to

(A) 1 (B) 0 (C) $\frac{1}{2}$ (D) 2

50. If $\sin\theta + \cos\theta = 1$, then the value of $\sin 2\theta$ is equal to

(A) 1 (B) $\frac{1}{2}$ (C) 0 (D) –1

51. If $\alpha + \beta = \frac{\pi}{4}$, then the value of $(1 + \tan\alpha)(1 + \tan\beta)$ is

(A) 1 (B) 2
(C) -2 (D) Not defined

52. If $\sin\theta = \frac{-4}{5}$ and θ lies in third quadrant then the value of $\cos\frac{\theta}{2}$ is

(A) $\frac{1}{5}$ (B) $-\frac{1}{\sqrt{10}}$ (C) $-\frac{1}{\sqrt{5}}$ (D) $\frac{1}{\sqrt{10}}$

53. Number of solutions of the equation $\tan x + \sec x = 2\cos x$ lying in the interval $[0, 2\pi]$ is

(A) 0 (B) 1 (C) 2 (D) 3

54. The value of $\sin\frac{\pi}{18}+\sin\frac{\pi}{9}+\sin\frac{2\pi}{9}+\sin\frac{5\pi}{18}$ is given by

(A) $\sin\frac{7\pi}{18}+\sin\frac{4\pi}{9}$ (B) 1

(C) $\cos\frac{\pi}{6}+\cos\frac{3\pi}{7}$ (D) $\cos\frac{\pi}{9}+\sin\frac{\pi}{9}$

55. If A lies in the second quadrant and 3 tan A + 4 = 0, then the value of 2 cotA − 5 cos A + sin A is equal to

(A) $\frac{-53}{10}$ (B) $\frac{23}{10}$ (C) $\frac{37}{10}$ (D) $\frac{7}{10}$

56. The value of $\cos^2 48° - \sin^2 12°$ is

(A) $\frac{\sqrt{5}+1}{8}$ (B) $\frac{\sqrt{5}-1}{8}$

(C) $\frac{\sqrt{5}+1}{5}$ (D) $\frac{\sqrt{5}+1}{2\sqrt{2}}$

[Hint: Use $\cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B)$]

57. If $\tan\alpha=\frac{1}{7}$, $\tan\beta=\frac{1}{3}$, then cos 2α is equal to

(A) sin 2β (B) sin 4β (C) sin 3β (D) cos 2β

58. If $\tan\theta=\frac{a}{b}$, then $b\cos 2\theta + a\sin 2\theta$ is equal to

(A) a (B) b (C) $\frac{a}{b}$ (D) None

59. If for real values of x, $\cos\theta = x+\frac{1}{x}$, then

(A) θ is an acute angle (B) θ is right angle

(C) θ is an obtuse angle (D) No value of θ is possible